

SUB-DOMAIN II EXPANSION AND FACTORIZATION

Exercise 2.4

STUDENTS LEARNING OUTCOMES

Recall the difference between

- Open and close sentences.
- Expression and equation.
- Equation and inequality.

Definition: A closed sentence is a statement that is either true or false, but not both.

Characteristics:

Complete: It provides a full idea or statement.

No Variables: It does not contain any placeholders or variables.

Truth Value: It can be evaluated as true or false.

Examples:

"The Earth orbits the Sun." (This is a factual statement that can be verified as true.)

"2 + 2 = 5." (This is a false statement.)

Definition: An open sentence is a statement that contains one or more variables and cannot be evaluated as true or false without assigning values to these variables.

Characteristics:

Variables: Contains placeholders that can be replaced by specific values.

Depends on Values: Truth value depends on the values assigned to the variables.

Incomplete Truth Evaluation: Cannot be evaluated as true or false until variables are defined.

Examples:

"x + 5 = 10" (The truth value depends on the value of x.)

"A person is happy if they are with friends." (The truth value depends on the context and what defines "happy" and "friends.")

Summary:

Closed Sentences: Complete and have a definite truth value.

Open Sentences: Contain variables and require additional information to determine their truth value.

Q.1: Separate the open and closed sentences.

(i) $3x + 4 = 1$

(ii) $2x^3 - 1 = 6$

(iii) $5 \times 4 = 20$

(iv) $5y + 7 = y$

(v) $10 + 40 = 50$

(vi) $72 + 8 = 9$

Solution:

Open sentences	Closed sentences
(i) $3x + 4 = 1$	(iii) $5 \times 4 = 20$
(ii) $2x^3 - 1 = 6$	(v) $10 + 40 = 50$
(iv) $5y + 7 = y$	(vi) $72 + 8 = 9$

Q.2: Separate the expressions and equations.

Expression: A combination of numbers, variables, and operators (like +, -, ×, ÷) without an equality sign. It represents a value but doesn't assert that two values are equal.

Equation: A statement that asserts the equality of two expressions, using an equality sign (=). It can be true or false depending on the values of the variables.

$$3x + 4 = 1$$

Expression: $3x + 4$

Equation: $3x + 4 = 1$

$$2x^3 - 1 = 6$$

Expression: $2x^3 - 1$

Equation: $2x^3 - 1 = 6$

Key fact

Expression:	Equation:
1. It is one sided.	1. It is two sided.
2. It has no relation symbol.	2. Its relation symbol is equality (=).

(i) $2x - 1 = 0$

(ii) $3x - y + 7$

(iii) $x + y = 3$

(iv) $7y^2 - 2y + 3 = 0$

(v) $x^2 - x - 1 = 0$

(vi) $x = -7$

Solution:

Expression	Equations
(ii) $3x - y + 7$	(i) $2x - 1 = 0$ (iii) $x + y = 3$
	(iv) $7y^2 - 2y + 3 = 0$
	(v) $x^2 - x - 1 = 0$

Q.3: Separate the equation and inequalities.

Equation: A statement that asserts the equality of two expressions, using an equality sign (=). It can be true or false depending on the values of the variables.

Inequality: A statement that compares two expressions using inequality signs (>, <, ≥, ≤, ≠).

> Greater Than

≥ Greater than or equal

< Less Than

≤ Less than or equal

≠ is not equal

$3x + 4 = 1$

- **Equation:** $3x + 4 = 1$

- **Not an Inequality:** This is not an inequality because it uses an equality sign.

$x + 5 > 10$

Not an Equation: This is an inequality because it uses the greater than sign (>).

i. $3x + 7 > 10$

Solution: Inequality

ii. $2x - 5 \leq 1$

Solution: inequality

iii. $x^2 + y + 7$

Solution: Equation

iv. $7x - 5 > 6x$

Solution: Inequality

v. $5x = 7$

Solution: Equation

vi. $\frac{10}{3} < 5$

Solution: Inequality

Q.4: Separate the polynomial expressions and expressions that are not polynomials.

Polynomial Expression: An algebraic expression involving a sum of powers in one or more variables multiplied by coefficients. The key features of polynomials are:

- Non-negative integer exponents on variables.

- Coefficients are real numbers.
- No variables in the denominator.
- No variables under radical signs.

Non-Polynomial Expression: An expression that does not meet the criteria of a polynomial. This includes:

- Negative or fractional exponents.
- Variables in the denominator.
- Variables under radical signs.
- Other complex functions like trigonometric functions or logarithms.

$3x + 4$

- **Polynomial Expression:** This is a polynomial. It is a first-degree polynomial (or linear polynomial) because it involves only non-negative integer exponents and has no variables in the denominator or radical signs.

$2/x + 4$

- **Not a Polynomial Expression:** This is not a polynomial because it includes a variable in the denominator.

(i) $x^2 + x - 1$

(ii) $x^2y + xy^2 + 7$

(iii) $x^{-2} + y + 7$

(iv) $\frac{x}{y^2} + 1 - \frac{y^2}{x}$

(v) $x^3 - x^2 + y - 1$

(vi) $x^4 + x^2 + 5x + \frac{1}{2}$

Solution:

Polynomial expressions	Non-Polynomial expressions
(i) $x^2 + x - 1$	(iii) $x^{-2} + y + 7$
(ii) $x^2y + xy^2 + 7$	
(v) $x^3 - x^2 + y - 1$	(iv) $\frac{x}{y^2} + 1 - \frac{y^2}{x}$
(vi) $x^4 + x^2 + 5x + \frac{1}{2}$	

Q.5: What constants are used in the following expressions?

Constant: A value that does not change. In algebraic expressions, constants are terms without variables and with fixed numerical values.

Combine with Variables: Constants can be combined with variables to form algebraic expressions. For example, in the expression $4x+7$, 4 is a constant coefficient of the variable x , and 7 is a constant term.

(i) $7x - 6y + 3z$

Solution: 7, -6 and 3 are constants.

(ii) $5x^2 - 3$

Solution: 5 and -3 are constants.

(iii) $8x^2 + 2y + 5$

Solution: 8, 2 and 5 are constants.

(iv) $9y + 3x - 2z$

Solution: 9, 3 and 2 are constants.

Q.6: Which the degree of the polynomials given below.

Identify the Polynomial:

- A polynomial is an algebraic expression consisting of terms that are sums or differences of variables raised to non-negative integer powers, multiplied by coefficients.

$$3x^2y^3+4xy^2-7x^3+5y-6$$

The degree of a term in a polynomial with multiple variables is the sum of the exponents of the variables in that term:

- $3x^2y^3$: The total degree is $2+3=5$.
- $4xy^2$: The total degree is $1+2=3$.
- $-7x^3$: The total degree is $3+0=3$.
- $5y$: The total degree is $0+1=1$.
- -6 : The total degree is $0+0=0$.

(i)	$x + 1$	Solution:	Degree =1
(ii)	$x^2 + x$	Solution	Degree = 2
(iii)	$x^3 - xy + 1$	Solution	Degree = 3
(iv)	$x^2y^2 + x^3 + y^2 - 1$	Solution	Degree = 4

Q.7: Separate the polynomials as linear, quadratic cubic and biquadratic.

Classification by Degree:

- **Linear Polynomial:** If the highest degree is 1.

Example 1:

$$P(x)=5x+3$$

1. Identify the Degree:

- The term $5x$ has an exponent of 1.
- The highest degree is 1.

2. Classification:

- This is a **linear polynomial** because the highest degree is 1.

- **Quadratic Polynomial:** If the highest degree is 2.

Example 2:

$$Q(x)=2x^2+4x+1$$

1. Identify the Degree:

- The term $2x^2$ has an exponent of 2.
- The highest degree is 2.

2. Classification:

- This is a **quadratic polynomial** because the highest degree is 2.

• **Cubic Polynomial:** If the highest degree is 3.

$$R(x) = x^3 - 3x^2 + 2x - 8$$

1. Identify the Degree:

- The term x^3 has an exponent of 3.
- The highest degree is 3.

2. Classification:

- This is a **cubic polynomial** because the highest degree is 3.

• **Biquadratic Polynomial:** If the highest degree is 4.

Example 4:

$$S(x) = x^4 - 2x^3 + x^2 + 7$$

1. Identify the Degree:

- The term x^4 has an exponent of 4.
- The highest degree is 4.

2. Classification:

- This is a **biquadratic polynomial** because the highest degree is 4.

(i) $3x + 1$

(ii) $x^2 - 2$

(iii) $y^2 - y$

(iv) $x + y$

(v) $x^3 - x^2 - 2$

(vi) $x^4 + x^3 + x^2$

(vii) $x^2y^2 + xy$

(viii) $x^2 + xy + 8$

(a) Linear (with degree 1) polynomials are:

(i) $3x + 1$

(iv) $x + y$

(b) Quadratic (with degree 2) polynomials are:

(ii) $x^2 - 2$

(iii) $y^2 - y$

(viii) $x^2 + xy + 8$

(c) Cubic (with degree 3) polynomials is.

(v) $x^3 - x^2 - 2$

(d) Biquadratic (with degree 4) polynomials are.

(vi) $x^4 + x^3 + x^2$

(vii) $x^2y^2 + xy$

Multiple Choice

1. The degree of the polynomial $x^3 + 4x + 5$ is.			
(A) 2	(B) 3	(C) 4	(D) 5
Explanation: The degree of a polynomial is the highest power of the variable in the polynomial. Here, the term with the highest power of x is x^3 , so the degree is 3.			
2. The polynomial $6x^3 - 4xy^2 + 7y + 1$ is.			
(A) Linear polynomial	(B) Quadratic polynomial	(C) Cubic polynomial	(D) Biquadratic polynomial
Explanation: The degree of a polynomial is determined by the highest degree of its terms. In this polynomial, the term $6x^3$ has the highest degree, which is 3. Hence, it is a cubic polynomial.			
3. The degree of polynomial $2x^3y^2z + 3x^4y^2z^2 + 6xy$ is.			
(A) 2	(B) 4	(C) 6	(D) 8
Explanation: The degree of a polynomial involving multiple variables is the highest sum of the exponents of the variables in any single term. For $2x^2y^2z$, the degree is $3 + 2 + 1 = 6$ for $3x^4y^2z^2$, it is $4 + 2 + 2 = 8$ and for $6xy$ it is $1 + 1 = 2$. Thus, the overall highest degree is 8.			
4. A closed sentence is always.			
(A) True	(B) False	(C) Open	(D) Both 'a' and 'b'
Reason: A closed sentence is a mathematical statement that is always true or always false.			
5. Which of the following is an example of an open sentence with two variables?			
(A) $x + 2 = 5$	(B) $2x + y = 7$	(C) $x - y = 3$	(D) Both 'b' and 'c'
Reason: An open sentence is an equation or inequality that contains variables and can be true or false depending on the values of the variables. Correct answer: Both 'b' and 'c'			
6. Degree of the polynomial $2x^4y^5$ is.			
(A) 4	(B) 5	(C) 7	(D) 9
Reason: The degree of a polynomial is the sum of the exponents of the variables in each term, and the highest degree among the terms is the degree of the polynomial. For $2x^4y^5$: Degree = $4 + 5 = 9$			
7. $x^2yz + xy^2z + xy + 1$ is example of			
(A) Polynomial in one variable	(B) Polynomials in two variable	(C) Polynomials in more variable	(D) All
Reason: This polynomial involves multiple variables (x , y , and z): Correct answer: Polynomials in more variables			
8. What is the degree of the polynomial $x^3 - 4x^2$ polynomials $(3x + 2)$ and $(2x - 1)$?			
(A) $6x^2 - x + 2$	(B) $6x^2 + 5x - 2$	(C) $6x^2 + x - 2$	(D) $6x^2 - 5x + 2$
The correct answer is $6x^2 - x + 2$, found by multiplying the polynomials $(3x + 2)$ and $(2x - 1)$.			
9. Polynomial $3y^2$ is.			
(A) Linear	(B) Quadratic	(C) Cubic	(D) Biquadratic
Reason: A quadratic polynomial is of the form $ay^2 + by + c$, where the highest power of the variable (in this case y) is 2. $3y^2$ fits this form.			

10. Which of the following is a closed sentence?			
Ⓐ $12 \div 4 = 3$	Ⓑ $x + 4 = 10$	Ⓒ $3y - 4 = 10$	Ⓓ $x + 2y = 7$
Reason: A closed sentence is a mathematical statement that is either true or false. $\frac{12}{4} = 3$ is a true statement. The other options involve variables and are not closed sentences since their truth value depends on the values of the variables.			
11. The algebraic expression is not polynomials.			
Ⓐ 13	Ⓑ $-x$	Ⓒ $\frac{1}{y}$	Ⓓ $5x + 3y$
Reason: A polynomial cannot have variables in the denominator. $1/y$ is not a polynomial because y is in the denominator.			
12. The polynomial xy^2z^2 has variable.			
Ⓐ 2	Ⓑ 3	Ⓒ 4	Ⓓ 5
Reason: The polynomial xy^2z^2 contains three different variables: x , y , and z . The answer is 3.			
13. The linear polynomial is.			
Ⓐ $x^4 + 4$	Ⓑ $y^3 + y^2$	Ⓒ $x^2 + 8$	Ⓓ $x + 2y$
Reason: A linear polynomial is of degree 1, meaning the highest power of any variable is 1. $x + 2y$ is linear because the highest power of x and y is 1.			
14. The degree of a cubic polynomial is.			
Ⓐ 1	Ⓑ 2	Ⓒ 3	Ⓓ 4
Reason: A cubic polynomial is defined as a polynomial of degree 3, meaning the highest power of the variable is 3.			